



RAINFALL-RUNOFF MODELING USING FUZZY TECHNIQUE FOR A SMALL WATERSHED IN MAHARASHTRA, INDIA

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ABSTRACT

In the present study, an effort has been made to develop fuzzy logic based runoff prediction model using current day's rainfall as input and daily runoff as output for a Harsul watershed of Godavari basin in the Nashik district of Maharashtra, India. The performance of the model was evaluated qualitatively and quantitatively by visual observation and employing various statistical indices viz. correlation coefficient, root mean square error, coefficient of efficiency, integral square error, coefficient of variation, mean absolute deviation and volumetric error. The values of these performance indicators for calibration (1993-2001) and validation (2002-2005) periods are 87.82 % and 90.26 %, 0.158 and 0.173, 77.16 % and 84.37 %, 0.0574 and 0.0433, 0.114 and 0.093, 1.71 and 1.30 and 9.51 % and 10.69 % respectively which are under acceptable limits. The performance of the model reveals that the model is able to predict the runoff with adequate accuracy.

KEYWORDS: Fuzzification, Root mean square error, Coefficient of Variation, Volumetric error,

INTRODUCTION

The hydrologic behavior of watershed in rainfall-runoff transformation process is very complicated phenomenon which is controlled by large number of climatic and physiographic factors that vary with both time and space. The relationship between rainfall and resulting runoff is quite complex and is influenced by factors relating the watershed and climate. Rainfall-runoff models are conventionally assigned to one of three broad categories: deterministic (physical), conceptual and parametric (also known as analytic or empirical) (Anderson and Burt, 1985 and Watts, 1997). Deterministic models describe the Rainfall-runoff process using physical laws of mass and energy transfer. Conceptual models provide simplified representations of key hydrological process using a perceived system (such as a series of interconnected stores and flow pathways). Parametric models use mathematical transfer functions (such as multiple linear regression equations) to relate meteorological variables to runoff. Hydrological models are further classified as either lumped or distributed (Todini, 1988). Lumped models treat the catchment as a single unit. They provide no information about the spatial distribution of inputs and outputs and simulate only the gross, spatially averaged response of the catchments. Conversely, distributed or heterogeneous models represent the catchment as a system of inter-related subsystems – both vertically and horizontally. Thus, distributed models can be considered as an assemblage of sub catchments arranged either in series or as a branched network (O'Loughlin *et al.*, 1996).

The need for accurate modeling of the rainfall-runoff process has grown rapidly in the past decades. However, considering the high stochastic property of the process, many models are still being developed in order to define such a complex phenomenon. Recently, Artificial Intelligence (AI) techniques such as the Artificial Neural Network (ANN) and the Adaptive Neural-Fuzzy Inference

System (ANFIS) have been extensively used by hydrologists for rainfall-runoff modeling as well as for other fields of hydrology. Some specific applications of ANN to hydrology include modeling rainfall-runoff process (Sajikumar *et al.*, 1999). A fuzzy rule-based system (FRBS) using the Takagi-Sugeno-Kang approach has been developed for Rainfall Runoff Modeling (Casper, *et al.* 2007). The Development of artificial neural network ANN and fuzzy logic FL models for predicting event based Rainfall runoff and tests these models against the kinematics wave approximation KWA (Tayfur and Singh, 2006).

Fuzzy set and fuzzy logic extend upon traditional Boolean logic (Zadeh, 1965). The fuzzy theory is for a mathematical description of imprecision and uncertainty in human experience and is used to reflect such complexities (Maskey *et al.*, 2004; Gopakumar and Mujumdar, 2008 and Chu and Chang, 2008). The concept of fuzzy logic modeling was originally proposed by Zadeh (1965), in which the linguistic variables rather than the numerical values are often used to facilitate the expression of rules and facts. Fuzzy logic modeling has been applied to various engineering problems in the past, e.g., a control of traffic junction, a water cleaning process, water level forecasting, stream flow prediction, and rainfall-runoff modeling (Chau *et al.* 2005; Şen and Altunkaynak 2006; Alvisi *et al.* 2006; Altunkaynak and Şen 2007; Özger 2009; Altunkaynak 2010). Comparison of Artificial Neural Network (ANN), Fuzzy Logic (FL) and linear transfer function (LTF)-based approaches for daily rainfall-runoff modeling (Lohani *et al.* 2011). A comparative case study between SWMM and fuzzy logic model for the predictions of total runoff with in the watershed of Cascina Scala, Pavia in Italy (Wang and Abdusselam, 2012).

In this study, an effort has been made to develop fuzzy logic based rainfall-runoff model of daily runoff prediction using current day's rainfall as input and daily

runoff as output for the Harsul watershed of Godavari basin in the Nashik district of Maharashtra, India with the following specific objectives, i) To generate and estimate daily runoff using developed model and ii) to assess the qualitative and quantitative performance of the developed model for the selected watershed.

MATERIALS AND METHODS

Description of Study Area

The Harsul watershed is situated under Godavari basin in the Nashik district of Maharashtra, India. It is situated between the longitudes 73° 25' E and 73° 29' E and the latitudes 20° 04' N and 20° 08' N. The watershed has a drainage area of 10.929 km². The highest peak in the area has an elevation 730 m above mean sea level, whereas the lowest point where the main stream drains has an elevation of 368 m above mean sea level, leading to an average slope of 6.6% along the main stream of the Harsul watershed. The soils in the watershed are mainly sandy silty loam. The watershed is mostly hilly and has undulating to rolling topography. The scattered and the sharp hills are intersected by numerous streams and gullies. The hills and the hill terraces of this area are mostly steep and rugged. The area can be divided into the five land forms namely hill terraces and plateau, hill slope and escarpment, subdued hills, valley and recent alluvium. The climate of the area in general is tropical and humid with three distinct seasons namely, monsoon, winter and summer. Mean annual precipitation in the watershed is 2275mm. The mean annual temperature is 26.5°C, minimum being 14°C in winter season and maximum being 36°C in summer season.

Collection of Hydrological Data

The daily rainfall and runoff data of monsoon season (1st June to 30th September) during period of 1993-2005 were obtained for Harsul watershed from State data storage centre of Water Resource Department, Nashik, Maharashtra.

Model Development

The Transformation of rainfall into runoff is highly complex, dynamic and nonlinear process which is affected by many factors which are often inter-related. The model development consists of system identification and parameter estimation.

Framework of Fuzzy Model

Zadeh (1965), a computer scientist, propounded the "fuzzy logic" or fuzzy set theory, based on the nature of fuzzy human thinking. Fuzzy Logic (FL) modelling refers to process whereby dynamical system is modeled not in the form of conventional differential and difference equations but in the form of set of fuzzy rules and corresponding membership functions. Fuzzy logic has been used as modelling methodology that allows easier translation between human and computers for decision making and better way to handle imprecise and uncertain information.

Fuzzy sets

Fuzzy sets are an extension of classical set theory and are used in fuzzy logic. In classical set theory the membership of elements, in relation to a set, is assessed in binary terms according to a crisp condition - an element either belongs or does not belong to the set. By contrast, fuzzy set theory permits the gradual assessment of the membership of

elements in relation to a set; this is described with the aid of a membership function $\mu = [0,1]$. Fuzzy sets are an extension of classical set theory since, for a certain universe, a membership function may act as an indicator function, mapping all elements to either 1 or 0, as in the classical notion.

Membership function

Every element in the universe of discourse is a member of a fuzzy set to some grade, maybe even zero. The set of elements that have a non-zero membership is called the support of the fuzzy set. The function that ties a number to each element of the universe is called the membership function.

Selection of input and output variables

Input and output variable are selected for the model on the basis of the study objectives. This is a crucial task for model development.

Input variable

The daily rainfall event is taken as input variable for the model. Therefore, only single input variable i.e. current day's rainfall (R_t) is applied for rainfall-runoff modelling of Harsul watershed.

Output variable

The daily runoff event or current day's runoff (O_t) is taken as output variable for fuzzy modelling of Harsul watershed. The input and output values are always a crisp numerical value limited to the universe of discourse of the input and output variables respectively. General flow diagram of fuzzy logic rule based algorithm is shown in Figure 1.

Fuzzification

Fuzzification is the process which converts each piece of input data to degree of membership by the lookup in one or several membership functions. The fuzzification block thus matches the input data with the conditions of the rules to determine how well the condition of each rule matches that particular input instance. There is a degree of membership for each linguistic term that applies to that input variable. The result of Fuzzification is called Fuzzy degree of membership, which varies in between 0 to 1.

In the present study, input (R_t) and output (O_t) were fuzzified into fuzzy subsets by using triangular membership functions in order to cover the whole range of changes. The criterion of defining fuzzy subsets is based on subjective perception of specific linguistic level by relevant experts. All inputs and output variables were separately divided into subsets, as extremely low (EL), very low (VL), low (L), medium (M), high (H), very high (VH) and extremely high (EH). More subsets are considered to increase the accuracy of prediction.

Formation of Fuzzy Rule Base

The rules may use several variables both in the condition and the conclusion of the rules. The Fuzzy rule base was formed based on the historical data and intuition. Fuzzy rule base contains Fuzzy rules that include all possible Fuzzy relations between inputs and output. These rules are expressed in the IF-THEN format. Fuzzy IF-THEN rules are conditional statements that describe the dependence of one linguistic variable on another. The analytical form of an IF-THEN is a fuzzy relation called the implication operation. For the case of missing historical values, the linguistic rules were developed, based on logic and

intuition or the data set with missing value was dropped from consideration.

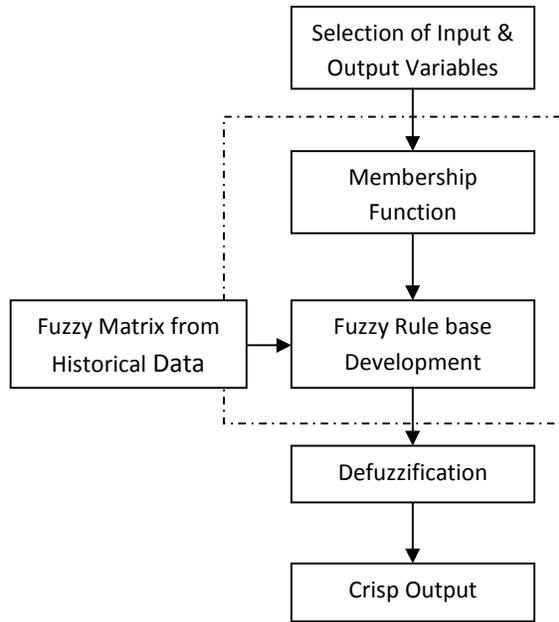


Fig. 1: Flow diagram of fuzzy logic rule based model Aggregation

Under aggregation, all of the fuzzy subsets assigned to each output variable are combined together to form a single fuzzy subset for each output variable. Aggregation is the unification of output of each rule by joining them. If an input value corresponds to both the membership functions, fuzzy rules corresponding to both the rules are invoked. Here, each rule invokes after implication, specifies one fuzzy output set. Then two fuzzy output sets are unified to get single output fuzzy set.

Defuzzification

Defuzzification is the process which converts the fuzzy value into a "crisp" value. Typically, a fuzzy system will have a number of rules that transform a number of variables into a "fuzzy" result, that is, the result is described in terms of membership in fuzzy sets. The result obtained from the implication is in the form of a fuzzy set. This is defuzzified to get a crisp output.

In the present study, the most common ‘centroid’ method of defuzzification was adopted. In the centroid method of defuzzification the real value is computed with the help of the following equation,

$$Cg = \frac{\sum_{i=1}^n y_i \times m_B(y_i)}{\sum_{i=1}^n m_B(y_i)} \dots\dots\dots (1)$$

where, Cg is the centroid of the truncated fuzzy output set B , $m_B(y_i)$ is the membership value of element y_i in the fuzzy output set B and n is the number of elements. In centroid method of defuzzification, all values of output were used.

Performance Evaluation of Model

Qualitative and quantitative evaluation of model is an essential task to assess their capability or potential of developed model in simulation of actual circumstances. In the present study the following qualitative and quantitative performance indices were applied to verify the applicability of developed model.

Quantitative evaluation

For better appreciation of the model, the predictive effectiveness of Fuzzy rule based model is judged on the basis of performance indicators. To judge the predictive capability of the developed model, correlation coefficient, root mean square error ((Wilks, 1995), coefficient of efficiency (Luchetta *et al.*, 2003), integral square error (Diskin *et al.*, 1978), coefficient of variation, mean absolute deviation (Yu *et al.*, 1994) and volumetric error were employed as shown in Table 1. The p_i is the predicted values, \bar{p}_{iN} is the mean of predicted values, a_i is the observed values, n is the number of observations and \bar{a}_{iN} is the mean of observed values.

RESULTS and DISCUSSION

The performance of the models was evaluated qualitatively and quantitatively by visual observation and employing various statistical indices viz. correlation coefficient, root mean square error, coefficient of efficiency, integral square error, coefficient of variation, mean absolute deviation and volumetric error. In this study, the acceptable limits for the correlation coefficient, coefficient of efficiency, and volumetric error have been considered to be above 75%, above 60% and less than 20% respectively. The daily prediction model for the study area has been developed with current day’s rainfall (R_t) as input and current day’s runoff (O_t) as the output for Harsul watershed. Input (R_t) and output (O_t) were fuzzified into fuzzy subsets. All inputs and output variables were separately divided into subsets, as extremely low (EL), very low (VL), low (L), medium (M), high (H), very high (VH) and extremely high (EH). More subsets are considered to increase the accuracy of prediction. In the defuzzification, the most common ‘centroid’ method of defuzzification was adopted. In the centroid method of defuzzification, the real value is computed with the help of Eqn. (1). The fuzzy model was constructed for the study watershed under fuzzy logic toolbox in soft computing programme MATLAB™ (R2008a).

Performance Evaluation of Developed Model

Qualitative evaluation

The qualitative evaluation of the model is based on the visual comparison, i.e., overall shape of the observed and predicted graphs. The qualitative assessment of models was made by regenerating daily runoff and by comparing the regenerated daily runoff with observed ones in order to verify and validate the equivalence between the catchment and model. Using fuzzy logic rule based model, the plots of observed and predicted values for Harsul watershed during the monsoon season (1st June to 30th September) from years 2002 to 2005 were depicted in Figs. 2 through 5. The plots show fair agreement between observed and predicted values.

Quantitative evaluation

The quantitative performance of developed model was also evaluated by applying various statistical indices. The values of above mentioned indices are presented in Table 2.

Correlation coefficient

The values of correlation coefficient for fuzzy logic rule based model were computed. Based on fuzzy logic rule model, the values of correlation coefficient for Harsul watershed for calibration (1993-2001) and validation (2002-2005) periods are 87.82 % and 90.26 % respectively. The higher values of correlation coefficient for training as well as testing periods show good agreement between observed and predicted values of runoff.

Root mean square error

The root mean square error (RMSE) values between observed and predicted values of runoff based on developed model of Harsul watershed for calibration and validation periods are 0.1583 and 0.1729 respectively.

Coefficient of efficiency

The coefficient of efficiency (CE), between observed and predicted values for asserting the applicability of fuzzy logic rule based model were determined and values so obtained are given in Table 2. Based on fuzzy logic rule model, the values of coefficient of efficiency of Harsul watershed for calibration and validation periods are 77.16% and 84.37% respectively.

Integral square error

The integral square error (ISE) is one of the commonly used measures to test goodness of fit of the developed model. The values of integral square error of study watershed for calibration and validation periods are 0.0574 and 0.0433 respectively.

Coefficient of variation

The coefficient of variation values between the ordinates of observed and predicted values of runoff were computed and are shown in Table 2. Applying the fuzzy logic rule based model, the values of coefficient of variation of Harsul watershed for calibration and validation periods are 0.114 and 0.093 respectively. The lower values of coefficient of variation suggest good agreement between computed and observed runoff.

Mean absolute deviation

For the fuzzy logic rule based model, the values of mean absolute deviation of Harsul watershed for calibration and validation periods are 1.71 and 1.30 respectively.

Volumetric error

The quantitative performance of the model was also assessed by another measure i.e. volumetric error and is given in Table 2. Using the fuzzy logic rule based model, the values of volumetric error of study watershed for calibration and validation period are 9.51 % and 10.69 % respectively.

CONCLUSIONS

The performance of Fuzzy Logic rule based model was found to be satisfactory on the basis of performance evaluation and can be applied for runoff prediction from study watershed.

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Table 1: Performance indicators

Indicator	Equation
Correlation coefficient (CC)	$CC = \frac{\sum_{i=1}^n \{(p_i - \overline{p_{iN}}) (a_i - \overline{a_{iN}})\}}{\sqrt{\sum_{i=1}^n (p_i - \overline{p_{iN}})^2} \sqrt{\sum_{i=1}^n (a_i - \overline{a_{iN}})^2}} \times 100\%$
Root mean square error (RMSE)	$RMSE = \sqrt{(1/n) \left(\sum_{i=1}^n (a_i - p_i)^2 \right)}$
Coefficient of efficiency (CE)	$CE = 1 - \frac{\sum_{i=1}^n (a_i - p_i)^2}{\sum_{i=1}^n (a_i - \overline{a_{iN}})^2} \times 100\%$
Integral square error (ISE)	$ISE = \frac{\sqrt{\sum_{i=1}^n (a_i - p_i)^2}}{\sum_{i=1}^n a_i}$
Coefficient of variation (CV)	$CV = \frac{\sqrt{\frac{1}{n} \sum_{i=1}^n (a_i - p_i)^2}}{\overline{Y}}$
Mean absolute deviation (MAD)	$MAD = \frac{1}{n} \sum_{i=1}^n p_i - a_i $
Volumetric error (VE)	$VE = \frac{\sum_{i=1}^n (p_i - a_i)}{\sum_{i=1}^n a_i} \times 100$

Table 2: Performance evaluation of the Fuzzy logic based model

Performance indices	Harsul watershed	
	Calibration Period	Validation Period
Root mean square error	0.1583	0.1719
Correlation coefficient	87.82 %	90.26 %
Coefficient of efficiency	77.16%	84.37%
Integral square error	0.0574	0.0433
Coefficient of variation	0.114	0.093
Mean absolute deviation	1.71	1.30
Volumetric error	9.51 %	10.69 %

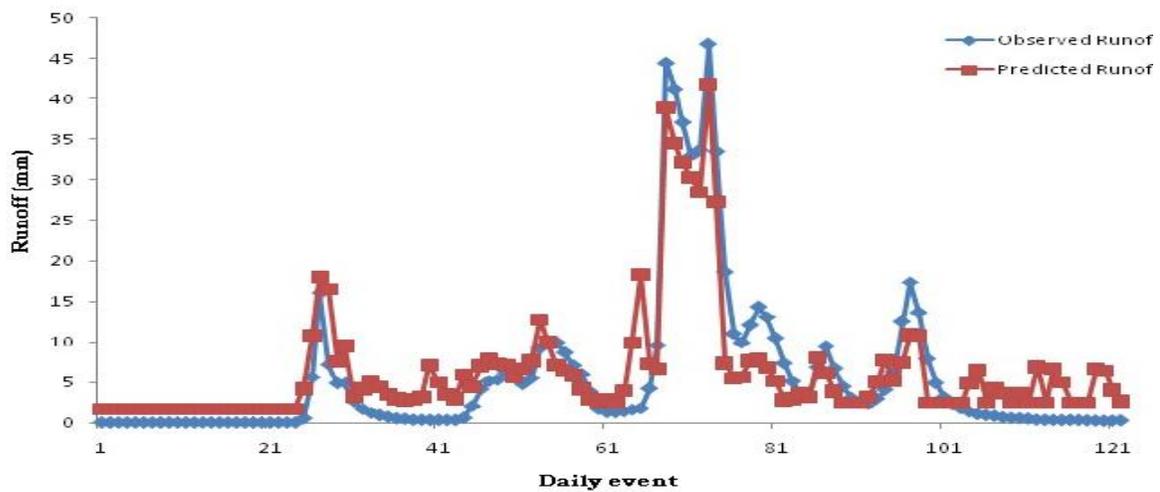


Fig. 2: Observed and predicted daily runoff using fuzzy logic model for Harsul watershed during active period of 2002

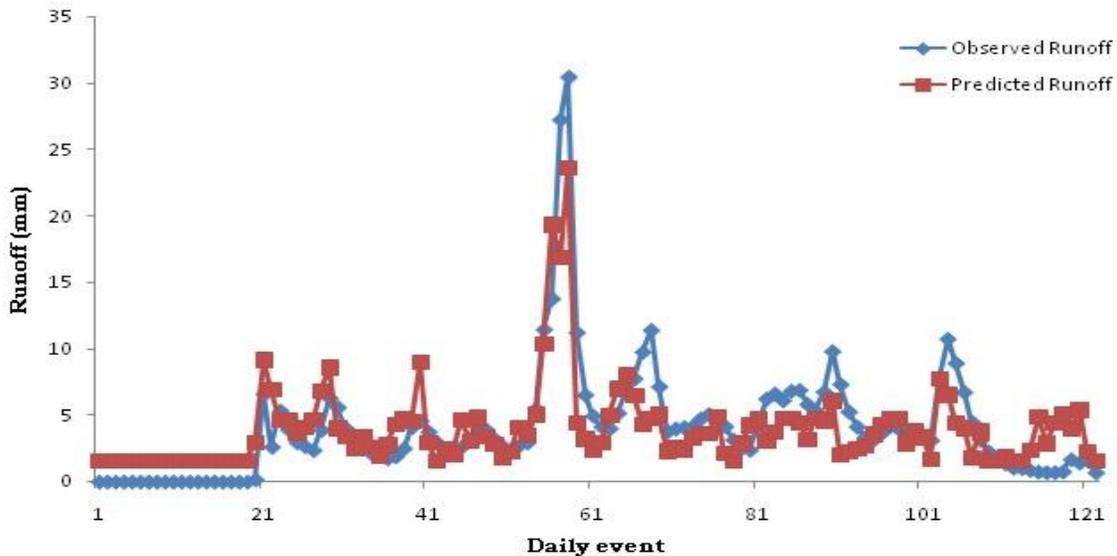


Fig. 3: Observed and predicted daily runoff using fuzzy logic model for Harsul watershed during active period of 2003

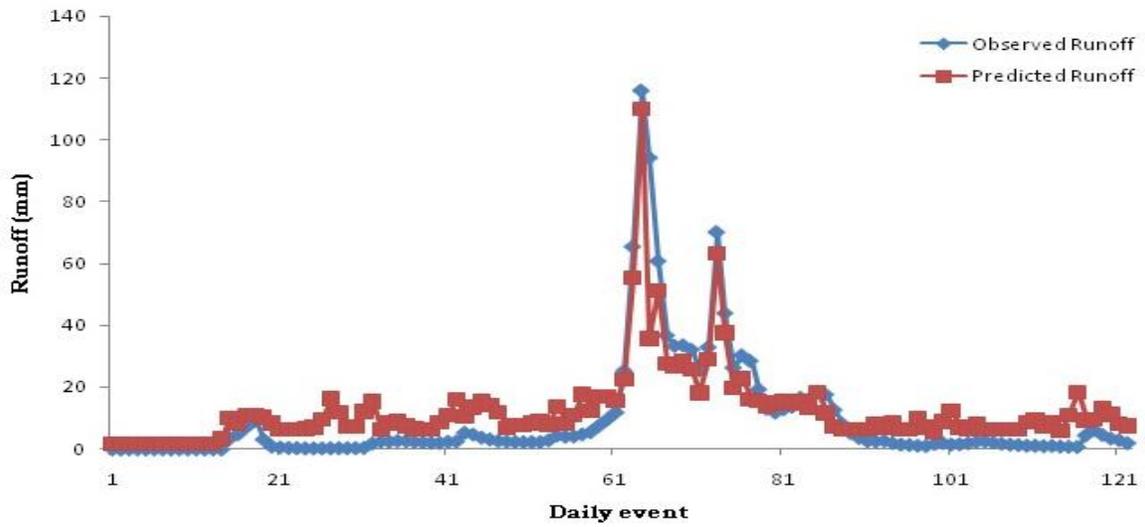


Fig. 4: Observed and predicted daily runoff using fuzzy logic model for Harsul watershed during active period of 2004

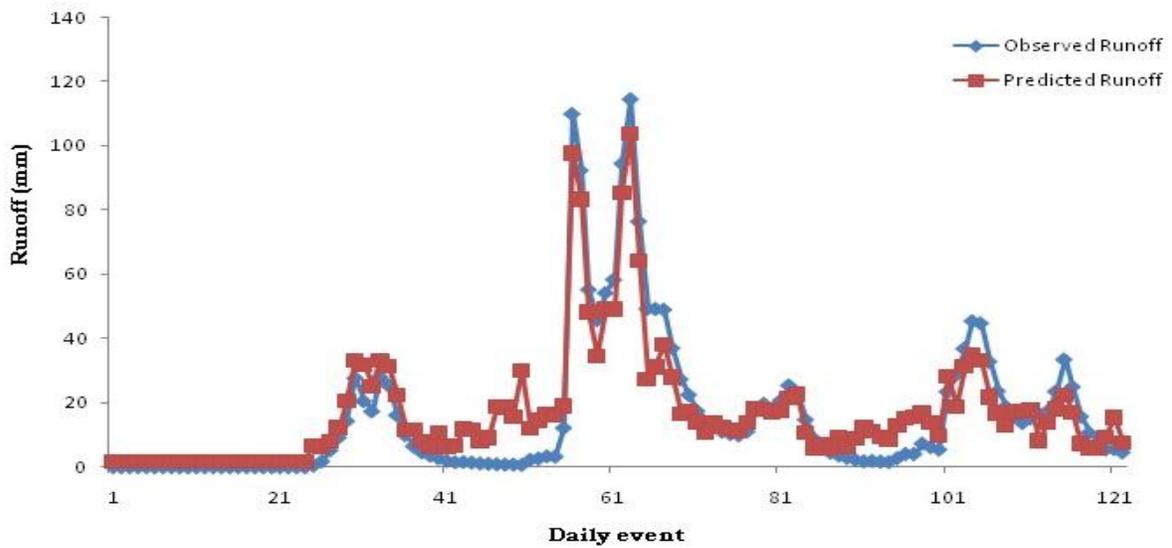


Fig. 5: Observed and predicted daily runoff using fuzzy logic model for Harsul watershed during active period of 2005