I.J.E.M.S., VOL.5 (1) 2014: 29-34 ISSN 2229-600X

INTERNATIONAL JOURNAL OF ENGINEERING AND MANAGEMENT SCIENCES

THE MODERN ANALYSIS OF THE RELIABILITY OF GPS NETWORKS

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ABSTRACT
A geodetic control network should be carefully designed, constructed, observed, adjusted, analyzed and maintained in order for it to provide a full lifetime of useful control. Complex and very precise geodetic networks may require expert analysis in order to realize the full accuracy and precision of the surveying observations used to create them. In addition to precision analysis, reliability analysis (the detection of outliers/blunders among the observations) has been measured using a technique pioneered by the geodesist Baarda. This work presents a research project on the modern analysis of the reliability of GPS Networks using different techniques and methods. In this study, commonly used conventional methods (statistical test) and the redundancy method have been applied to two (2) GPS Networks with different characteristics. The GPS baseline components have been taken as the measurements. The GPS networks were first adjusted using the MATRIX program and latter analyzed using the ADJUST program. The conventional methods (i.e. Data Snooping and Tau Test) were applied to the networks at different significant levels. In GPS networks with redundant observations, choosing the significant level as 0.001 was sufficient to realize outlier detection procedure. Working with great significant levels produced unreliable results. For the redundancy number, the global relative redundancy was calculated. It was seen that an adjustment that in general has low redundancy numbers will have measurement that lack sufficient checks to isolate blunders, and thus the chance for undetected blunders to exist in the measurements is high. Conversely, high overall redundancy number enables a high level of internal measurement checking and thus there is a lower chance of accepting measurements that contain blunders. Reliability theory proves to be an effective method for network monitoring and a useful design tool when applied to GPS geodetic control networks. It should be considered when designing GPS control networks.

KEYWORDS: Geodetic/GPS Control Network, Baseline Components, Reliability Analysis, Conventional methods, Redundancy number.

1. INTRODUCTION

1.1 Geodetic Control Network

A geodetic control network is the wire-frame or the skeleton on which continuous and consistent mapping, Geographic Information Systems (GIS), and surveys are based. To understand the function of geodetic control, we have to realize that a map or a plane survey is a flat representation of the curved world. If we want the maps to become an authentic representation of the real world, we have to be able to "paste" small pieces of (flat) map contents onto a curved world.

Traditionally, geodetic control points are established as permanent physical monuments placed in the ground and precisely marked, located, and documented. Locating spatial features with respect to geodetic control enables the accuracy assessment of these features. Interest and activity regarding geodetic control has dramatically increased at all government levels because of the need for accurate maps and surveys used in geographic and land information systems.

With the advent of the Global Positioning System (GPS), the framework of the geodetic control network should preferably be based on CORS (Continuously Operating Reference Stations). CORS stations provide an active geodetic control network, which enable GPS users to tie their positioning observations to the geodetic network without physically having to occupy a geodetic control point.

GPS (Global Positioning System) devices can be used in many applications which require accurate point positioning in geosciences. Complex and very precise geodetic networks may require expert analysis in order to realize the full accuracy and precision of the surveying observations used to create them. Accuracy of GPS decreases due to outliers resulting from the errors inherent in GPS observations. Outliers should be detected and eliminated from the adjustment in other for it not to affect the rest of the observations. Several approaches have been developed to detect outliers in geodetic observations. It is also important to determine which method is most effective at distinguishing outliers from normal observations.

2. METHODOLOGY

2.1 Data Requirement

The data used for this research were grouped into two main classes; Primary and Secondary Data sets. For the purpose of this research and to focus on GPS Network Analysis; Secondary Data was used. The GPS Baseline Components ΔX, ΔY, ΔZ and the Variance-Covariance Matrix of the Network will be taken as observations.

2.2 Data Acquisition

The data used for this research was the GPS Baseline Components and the Variance-Covariance Matrix of the Network. The properties of the Network include; Number of the Points, Number of the Baseline, Number of the Observation (n), Number of the Unknowns u (parameter), Datum Defects (d), Redundant Observation (f = n – u + d), Number of the Triangles (n_t), Covariance Matrix.

2.3 Data Processing

In application of Conventional Methods, the Significant Level of Test Statistics was obtained from the equation $\alpha_0 = 1 - (1 - \alpha)^{1/n} = \alpha / n$ as 0.001 for First GPS Network and 0.002 for the Second GPS network. Where α is the Total Significant Level and usually chosen as 5%, n is the number of observations. Since such a Significant Level (i.e. 0.002) may cause insensitivity for outliers, it has been chosen as 0.001 in Statistical Testing and it has been seen that when the Significant Level gets greater more outliers appears. The results of the Significant Levels 0.05, 0.01 and 0.001 are presented in this study.

2.3.1 The Matrix Program

First a data file (Fig. 1) containing the coefficient matrix (A), weight matrix (W), and constants matrix (L) was created and selected from the list of matrices read. Using the weighted observation equation WAX = WL for the linear system of equations, or J, W, and K for the nonlinear system of equations WJX = WK.

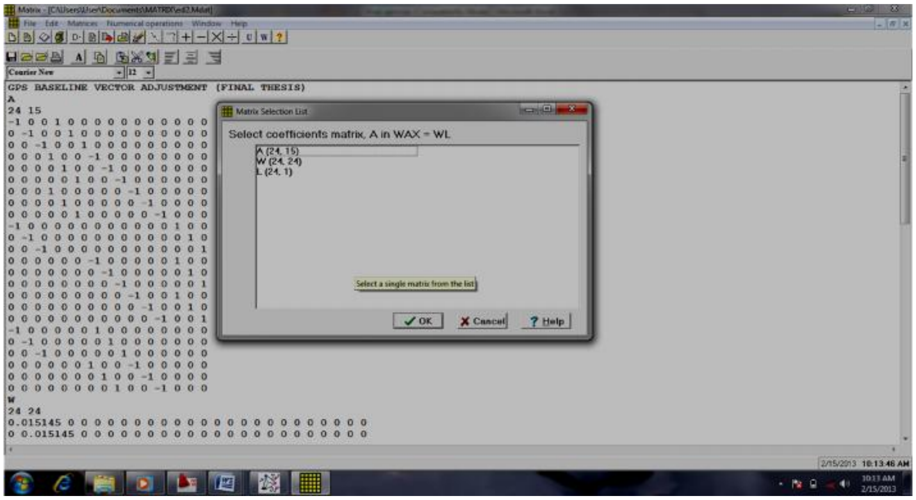

Figure 1: Data File for Least squares adjustment using Matrix Program

2.3.2 The Adjust Program

The data file (Fig. 2) was created using the format below.

File format:

Title Line: e.g. Job number 123

#-Control Station #-unknown stations #-baseline vectors

List of control station coordinates:

ID, X, Y, Z

List of GPS distance vectors:

From To dx dy dz Sxx Sxy Sxz Syy Syz Szz

The options for the network adjustment were; printing of the A, L and other matrices to a file with the name {data file name .mat}, creating a file of adjusted geocentric coordinates with the name {data file name}.xyz and computing adjusted observational standard deviations

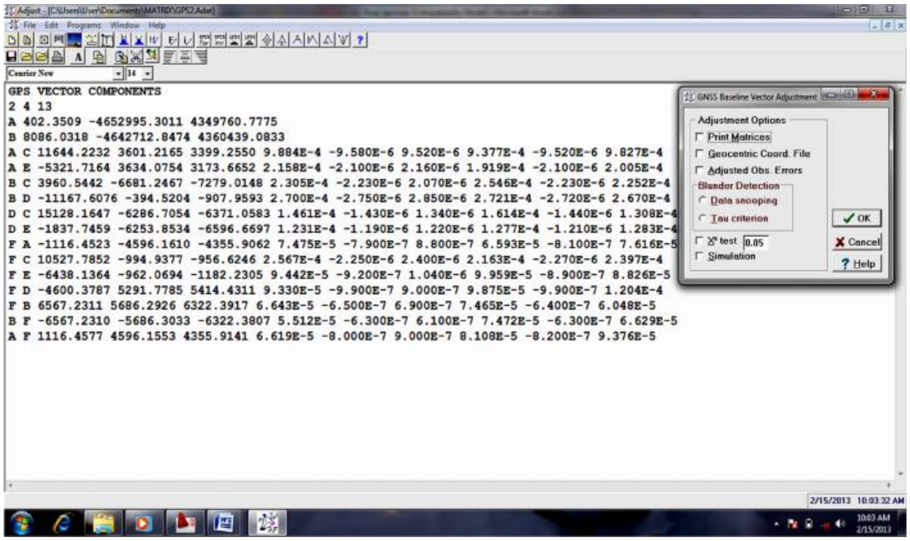

Figure 2: Data File for GPS baseline vector adjustment using Adjust Program

3. DATA ANALYSIS

In this study, two (2) GPS Networks were evaluated to examine for outliers using different methods. In order to focus on only the networks and not on the external constraints, a free adjustment strategy has been applied. The properties of the networks are listed below:

Table 1: Properties of the Two (2) GPS Networks

INFORMATION ABOUT NETWORKS	FIRST NETWORK	SECOND NETWORK
- Number of Points	6	5
- Number of Baseline	13	8
- Number of Observations (n)	39	24
- Number of unknowns (u)	12	15
- Datum Defect (d)	3	3
- Redundant Observations (f = n-u+d)	30	12
- Number of Triangles (n_t)	26	10
- A priori Standard Deviation (S_0)mm	1	1

Table 2: GPS Vector Components of the Network

Station		X		Y	Z
A		402.3509		-4652995.3011	4349760.7775
B		8086.0318		4642712.8474	4360439.0833
Distance Vectors					
FROM	**TO**	δx	δy	δz	**COVARIANCE MATRIX ELEMENTS**
A	C	11644.2232	3601.2165	3399.2550	9.8E-4 -9.5E-6 9.5E-6 9.3E-4 -9.5E-6 9.8E-4
A	E	-5321.7164	3634.0754	3173.6652	2.1E-4 -2.1E-6 2.1E-6 1.9E-4 -2.1E-6 2.0E-4
B	C	3960.5442	-6681.2467	-7279.0148	2.3E-4 -2.2E-6 2.0E-6 2.5E-4 -2.2E-6 2.2E-4
B	D	-11167.6076	-394.5204	-907.9593	2.7E-4 -2.7E-6 2.8E-6 2.7E-4 -2.7E-6 2.6E-4
D	C	15128.1647	-6286.7054	-6371.0583	1.4E-4 -1.4E-6 1.3E-6 1.6E-4 -1.4E-6 1.3E-4
D	E	-1837.7459	-6253.8534	-6596.6697	1.2E-4 -1.1E-6 1.2E-6 1.2E-4 -1.2E-6 1.2E-4
F	A	-1116.4523	-4596.1610	-4355.9062	7.4E-5 -7.9E-7 8.8E-7 6.5E-5 -8.1E-7 7.6E-5
F	C	10527.7852	-994.9377	-956.6246	2.5E-4 -2.2E-6 2.4E-6 2.1E-4 -2.2E-6 2.3E-4
F	E	-6438.1364	-962.0694	-1182.2305	9.4E-5 -9.2E-7 1.0E-6 9.9E-5 -8.9E-7 8.8E-5
F	D	-4600.3787	5291.7785	5414.4311	9.3E-5 -9.9E-7 9.0E-7 9.8E-5 -9.9E-7 1.2E-4
F	B	6567.2311	5686.2926	6322.3917	6.6E-5 -6.5E-7 6.9E-7 7.4E-5 -6.4E-7 6.0E-5
B	F	-6567.2310	-5686.3033	-6322.3807	5.5E-5 -6.3E-7 6.1E-7 7.4E-5 -6.3E-7 6.6E-5
A	F	1116 .4577	4596.1553	4355.9141	6.6E-5 -8.0E-7 9.0E-7 8.1E-5 -8.2E-7 9.3E-5
A	B	7683.6883	10282.455	10678.301	7.2E-4 -7.3E-6 7.5E-6 6.8E-4 -7.3E-6 7.3E-4

Table 3: Second GPS Network: Distance Vectors

FROM	TO	δx	δy	δz	COVARIANCE MATRIX ELEMENTS					
1	2	-4686.615	-17721.671	-14884.397	1.5E-2	0	0	1.5E-2	0	1.5E-2
3	2	8499.814	-5663.546	-8403.435	5.0E-3	0	0	5.0E-3	0	5.0E-3
4	2	14054.892	1236.850	-6290.594	6.7E-3	0	0	6.7E-3	0	6.7E-3
1	5	-20861.369	-3409.884	4406.536	1.2E-2	0	0	1.2E-2	0	1.2E-2
3	5	-7675.107	8647.889	10887.576	7.1E-3	0	0	7.1E-3	0	7.1E-3
4	5	-2119.837	13074.044	13001.240	9.5E-3	0	0	9.5E-3	0	9.5E-3
1	3	-13185.916	-12059.053	-6479.730	1.0E-2	0	0	1.0E-2	0	1.0E-2
4	3	5554.984	4427.085	2112.415	1.7E-3	0	0	1.7E-3	0	1.7E-3

3.1 First GPS Network

The First GPS Network consists of two control stations and four stations whose coordinates were to be determined. There have the following NAD83 geodetic and geocentric coordinates:-

φ_A = 43°15’46.2890”	X_A = 402.3509m	λ_A = -89°59’42.1640”
Y_A = -4652995.3011m	h_A = 1382.618m	Z_A = 4349760.7775m
φ_B = 43°23’46.3626”	X_B = 8086.0318m	λ_B = -89°54’00.7570”
Y_B = -4642712.8474	h_B = 1235.457m	Z_{AB}= 4360439.0833m

A summary of the baseline measurements obtained from the Least-Squares Adjustment of carrier-phase observations for this Network is given in Table 2. The Covariance Matrix elements that are listed in Table 2 were used for weighting the observations. A network adjustment yielded adjusted X coordinates for the stations (and adjusted coordinate differences between stations) that were all mutually consistent. Specifically for this network, the adjusted X coordinate station A; and the same value was obtained by adding ΔX_{AC} to the X coordinate station

A; and the same value was obtained by adding ΔX_{BC} to the X coordinate of Station D and so on. Equivalent conditions existed for the Y and Z coordinates. Note that these conditions did not exist for the data of Table 2, which contained the unadjusted baseline measurements.

3.2 Network Preadjustment Data Analysis

3.2.1 Analysis of Fixed Baseline Measurements

Note that in the data of Table 2, one fixed baseline (between control points A and B) was measured. The fixed baseline was used only for checking, but not included in adjustment. Table 4 gives the data for comparing the measured and fixed baseline components. The measured values are listed in column (2) and the fixed components are given in column (3). To compute the fixed values, X_e, Y_e, Z_e Geocentric Coordinates of the two Control Stations were first determined from their Geodetic Coordinates. Then the ΔX, ΔY and ΔZ differences between the X_e, Y_e, Z_e coordinates for two Control Stations were determined. Differences (in meters) between the measured and fixed baseline components are given in column (4). Finally the differences, expressed in parts per million (ppm), are listed in column (5). These ppm values were obtained by dividing column (4) differences by their corresponding total baseline lengths and multiplying by 1,000,000.

3.2.2 Analysis of Repeat Baseline Measurement

In the data of Table 2, baselines AF and BF were repeated. Table 5 gives comparisons of these measurements using the same procedure that was used in Table 4. Again, the ppm values listed in column (5) used the total baseline lengths in the denominator which was computed from the square root of the sum of the squares of the measured baseline components.

Table 4: Comparisons of Measured and Fixed Baseline Components.

COMPONENT (1)	MEASURED (m) (2)	FIXED (m) (3)	DIFFERENCE (4)	PPM (5)
ΔX	7,683,6883	7,683.6809	0.0074	0.44
ΔY	10,282.4550	10,282.4537	0.0013	0.08
ΔZ	10,678,3008	10,678.3058	0.0050	0.30

Table 5: Comparisons of Repeated Baseline Measurements.

Component (1)	First Observation (2)	Second Observation (3)	Difference (4)	PPM (5)
ΔX_{AF}	1116.4577	-1116.4523	0.0054	0.84
ΔY_{AF}	4596.1553	-4596.1610	0.0057	0.88
ΔZ_{AF}	4355.9141	-4355.9062	0.0079	1.23
ΔX_{BF}	-6567.2310	6567.2311	0.0001	0.01
ΔY_{BF}	-5686.3033	5686.2926	0.0107	1.00
ΔZ_{BF}	-6322.3807	6322.3917	0.0110	1.02

3.3 Analysis of Loop Closures

GPS networks will typically consist of many interconnected closed loops. In the network of Table 2, a closed loop is formed by points ACBDEA. Similarly, ACFA, CFBC, BDFB, and so on, are other closed loops. For each closed loop, the algebraic sum of the ΔX components should equal zero. The same condition should exist for the ΔY and ΔZ components. To compute loop closures, the baseline components were simply added algebraically for that loop. The closure in X components for loop ACBDEA, for example, would be computed as $cx = \Delta X_{AC} + \Delta X_{CB} + \Delta X_{BD} + \Delta X_{DE} + \Delta X_{EA}$ where cx is the loop closure in X coordinates. Similar equations apply for computing closures in Y and Z coordinates.

$$cx = \Delta X_{AC} + \Delta X_{CB} + \Delta X_{BD} + \Delta X_{DE} + \Delta X_{EA}$$

Where cx is the loop closure in X coordinates. Similar equations apply for computing closures in Y and Z coordinates. Substituting numerical values into Equation (13), the closure in X coordinates for loop ACBDEA is:

$cx = 11{,}644.2232 - 3960.5442 - 11{,}167.6076 - 1837.7459 + 5321.7164 = 0.0419m$

Similarly, closures in Y and Z coordinates for that loop are

$cy = 3601.2165 + 6681.2467 - 394.5204 - 6253.8534 - 3634.0754 = 0.0140m$

$cz = 3399.2550 + 7279.0148 - 907.9593 - 6596.6697 - 3173.6652 = -0.0244m$

For loop ACBDEA the resultant is 0.0505m. For loop ACBDEA, the total loop length is 50,967m, and the closure ppm ratio is therefore (0.0505/50,967) (1,000,000) = 0.99 ppm.

The results of the output file of the Matrix Program for the adjustment of the Network includes; Design Matrix (A), Misclosure Matrix (L), Weight Matrix (W), N & Qxx Matrix, X & V(residuals) Matrix, $S0^2$ & Sxx Matrix.

3.4 Second GPS Network

The Second GPS Network (Fig. 3) was adjusted using the Matrix Program. The weight matrix for the vectors are block diagonal with the correlations reflecting satellite geometry. Because vectors are oriented, the minimal constraints are imposed by fixing one station. The inner constraint solution does have the advantage that the standard ellipses or standard ellipsoids are independent of the definition of the coordinate system. The outcome of the inner constraint solution is summarized in Table 8. The vector observations are reduced to the station marker during the adjustment.

The results of the output file for the Design Matrix (A), Misclosure Matrix (L), Weight Matrix (W), N & Qxx Matrices, X & V (residuals) Matrices, $S0^2$ & Sxx Matrices were shown. Outlier detection analysis was done using the Adjust Program.

3.5 Conventional Methods

The Matrix and Adjust Program were used in the adjustment and reliability analysis.

3.5.1 Tau Test and Data Snooping

During Network Adjustment, the results obtained from the two (2) programs were examined for comparative analysis. For the First GPS Network, the ADJUST program had its Degrees of Freedom = 27, Reference Variance = 0.5015 and Standard Deviation of Unit Weight = ±0.71. The MATRIX program's Degrees of Freedom = 27, Reference Variance = 0.6149 and Standard Deviation of Unit Weight = ±0.78 (Table 6 & 7). The Second GPS is as contained in Table 8.

The proper procedure for removing blunders is to remove the single observation which is greater in magnitude than the rejection level selected for the adjustment and is also greater in magnitude than the value of any other standardized residual in the adjustment. This procedure prevents removing observations that are connected to blunders and thus are inherently affected by their presence.

3.6 Redundancy Method

For uniform network strength, the individual redundancy number r should be close to the global relative redundancy of r/m, where r is the number of redundancy measurements and m is the number of measurements in the network. Weak areas in the network are located by finding regions where the redundancy numbers become small in comparison to relative redundancy. For the First GPS Network, r = 30 and m = 39 therefore the global relative redundancy r/m = 0.8. Also, for the Second GPS Network, r = 12 and m = 24 therefore the global relative redundancy r/m = 0.5. The redundancy numbers should be greater than 0.8 and 0.5 respectively. When considering the two GPS networks, few slightly weak areas of the networks were located at regions were the redundancy numbers were less that the relative redundancy, but not enough to be masked or considered as outliers.

4. PRESENTATION OF RESULTS

In the conventional methods, when comparing the values of the standardized residuals against the rejection level of the adjustment, it was seen that there were no possible blunders since the test statistics of the standardized residuals are less than the critical level. Thus no measurement was removed from the observations. On the event of removing of measurement from the observations the adjustment will be rerun. In the Conventional Methods, no outlier has been detected at a significance level of 0.01 and 0.001 for the First and Second GPS network. Hence, the Statistical Test has not been applied at a significance level of 0.001. The values of the redundancy numbers obtained from the analysis of the networks when compared to the relative redundancy number also showed a good indication of geometrically strengthened networks.

Table 6: Results of the Conventional Methods for the Two (2) GPS Networks (Significant Level = 0.001).

Statistical	Test	Significant Level	Maximum Test Statistic	Critical Value	Outlier
First GPS Network	Tau	0.001	2.603	3.675	-
	DS	0.001	2.330	3.290	-
Second GPS Network	Tau	0.001	2.379	2.519	-
	DS	0.001	1.532	2.581	-

Table 7: Adjusted Coordinates of the First GPS Network.

STATION	X	Y	Z	Sx	Sy	Sz
A	402.3509	-4,652,995.3011	4,349,760.7775			
B	8,086.0318	-4,642,712.8474	4,360,439.0833			
C	12,046.5808	-4,649,394.0826	4,353,160.0644	0.0086	0.0087	0.0084
D	-4,919.3391	-4,649,361.2199	4,352,934.4548	0.0074	0.0074	0.0073
E	-3,081.5831	-4,643,107.3692	4,359,531.1233	0.0070	0.0072	0.0073
F	1,518.8012	-4,648,399.1453	4,354,116.6914	0.0038	0.0040	0.0040
Degrees of Freedom = 27		Reference Variance = 0.5015		Standard Deviation of Unit Weight = ±0.71		

Table 8: Inner Constraint GPS Vector Adjustment of the Second GPS Network**.**

St	St	Observation mark to mark	Residual	Adj. obs (m)
1	**2**	-4686.432	0.108	-4686.324
		-17722.135	0.046	-17722.088
3	**2**	-14883.891	0.087	-14883.805
		8499.881	-0.042	8499.839
4	**2**	-5663.722	-0.001	-5663.724
		-8403.242	-0.032	-8403.273
		14054.939	0.008	14054.947
1	**5**	-1236.981	-0.019	-1237.000
		-6290.451	0.004	-6290.447
		-20861.191	-0.135	-20861.326
3	**5**	-3410.326	0.028	-3410.298
		4407.007	-0.055	4406.952
		-7675.164	0.001	-7675.163

4	**5**	8648.038	0.029	8648.067
		10887.410	0.073	10887.483
		-2120.155	0.100	-2120.055
1	**3**	13074.850	-0.060	13074.790
		13000.367	-0.057	13000.310
		-13186.196	0.034	-13186.162
4	**3**	-12058.312	-0.053	-12058.365
		-6480.517	-0.014	-6480.531
		5555.129	-0.021	5555.108
		4426.708	0.016	4426.724
		2112.817	0.010	2112.826

5. CONCLUSION

The optimization and design of geodetic networks is an important part of most geodesy projects, which are carried out before the measurements are actually made. This not only results in an optimal design, but also will save much of the time and of the cost. One of the criteria described to characterize the quality of a geodetic network is the reliability and geometrical strength. A geodetic network is of high reliability if the smallest possible gross errors can be detected. To obtain reliable results the networks has to be adjusted and analyzed. In GPS networks with redundant observations, choosing the significant level as 0.001 was sufficient to realize outlier detection procedure. Working with great significant levels produces unreliable results. In conventional methods, a normal observation may seem as outlier at the end of iterations and may be removed from observation set. Thus, the shape of the network is defected. In this study, it was seen that it is appropriate to apply conventional detection tests at significance level of 0.001 in GPS networks. If the conventional methods are used at very small significance levels, these methods tend to mask the outliers. On the other hand, at greater significance level such as 0.01, more outliers will appear. So, the significant level can be selected as 0.001 in GPS networks that have too many observations. In the first and second GPS networks, there appeared no outliers at any significant level. In network design, one should always check the redundancy numbers of the anticipated observation and strive to achieve a uniformly high value for all observations. To reach this goal, one can maximize the redundancy numbers (usually the smallest one) of the observations. Redundancy numbers generally are sufficient to provide well-checked measurement.

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