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EFFECTIVENESS OF MULTI-COLLINEARITY PROBLEM FOR ESTIMATING FUZZY LINEAR REGRESSION PARAMETERS WITH APPLICATION

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ABSTRACT
Fatigue cracking or alligator cracking is a common type of distress in asphalt pavement. There are many reasons that cause the fatigue cracking in asphalt pavements such of asphalt mixtures which consist of asphalt, aggregate and mineral filler. Fatigue cracking distress in asphalt pavements most often instigated by failure of the surface due to traffic loading. However, it can be greatly influenced by environmental and other effects while traffic loading remains the direct cause. Frequently, overloading happens because the base or sub base inadequately support the surface layer and subsequently cannot handle loads that it would normally endure. In this research many factors were used to build a fuzzy linear regression model like fatigue life, initial flexural stiffness modulus, and initial tensile strain, stress level and air voids. The model had been analyzed by using the classical technique and our proposed procedure. The total spread error was used to compare the performance of the calculated procedures.

KEYWORDS: Fuzzy triangular numbers; Fuzzy linear regression; principle component method; ridge regression method and linear programming.

INTRODUCTION

The fuzzy linear regression was proposed by Tanaka et al (1982). This model is applied on different problems in various life patterns like physics, chemistry, engineering etc., but it has to be considered that it contains uncertainty and vagueness in its data. Even though, it looks the linear multi-collinearity problem is ignored in solving problems, and that neglecting leads to inaccurate or not true estimators of fuzzy parameters. That means the extrusive effect becomes negative, In general, the popular methods to deal with linear multi-collinearity problem in traditional statistics are ridge regression and principle component regression [1]. On the other hand the two authors (Savic & Pedrycz) express the fuzzy linear regression problems to minimize the total spread values which is valid when the utilized data are crisp inputs and fuzzy output, and their method constitute two steps: the first step determines minimal vagueness through least squares and then utilizing the results from the first step to yield the spread values through linear programing in the second step.

In this research, two approaches were suggested to reduce the total spread value when the given data suffers from linear multi-collinearity problem, and these are:

The first proposed approach by using principle component regression and the second proposed approach by using ridge regression.

A numerical example is used to describe the properties of the estimators of fuzzy regression parameters obtained by using the proposed approaches when the data suffer from a near multi-collinearity problem.

Fuzzy linear regression (FLR)

Fuzzy linear regression (FLR) is a fuzzy type of classical regression analysis in which some elements of the model are represented by fuzzy numbers. It is used in evaluating the functional relationship between the dependent and independent variables in a fuzzy environment, so if the used function was linear then that is called fuzzy linear regression. While the traditional regression is based on the probability theory, the fuzzy linear regression is based on fuzzy sets theory.

The fuzzy sets theory is a suitable tool to deal with uncertainty. Which contain linguistic data or data that constitute non-fixed error. Various studies were conducted trying to merge the statistical methods and fuzzy sets theory.

Fuzzy linear model is categorized according to variables into three types [2]:

1. Inputs and outputs, both are non-fuzzy (crisp) numbers.
2. Inputs are non-fuzzy (crisp) while the outputs are fuzzy numbers.
3. Inputs and outputs are both fuzzy numbers.

Fuzzy regression methods

In this article, different approaches for analyzing fuzzy linear regression are presented. Moreover, our contribution to the original Savic & Pedrycz approach is illustrated.

1. Tanaka's model

In (FLR) analysis, some assumptions concerned traditional regression analyses are relaxed and the uncertainty is represented by a fuzzy relationship between the input and output.

The present paper considers first the model of Tanaka which is a pioneer for such models.

The basic Tanaka' model assumes a linear relation:

$$\tilde{y}_i = \tilde{\beta}_0 + \tilde{\beta}_1 X_{i1} + \tilde{\beta}_2 X_{i2} + \cdots + +\tilde{\beta}_p X_{ip} \,,\ for\ i = 1,2,\ldots,n \tag{1}$$

$$(y_i, e_i) = (\alpha_0, c_0) + (\alpha_1, c_1) X_{i1} + (\alpha_2, c_2) X_{i2} + \cdots + +(\alpha_p, c_p) X_{ip} \tag{2}$$

Where:

$\tilde{y}_i = (y_i, e_i)$ is the symmetric fuzzy triangular output number where y_i represent the center and e_i is the left-right spread.

$\tilde{\beta}_j = (\alpha_j, c_j)$ is the symmetric fuzzy coefficient of triangular number where α_j represent the center and c_j is the left-right spread, $c_j \geq 0$, $j = 1, 2, \dots, p$.

$x_i = (x_{i1}, x_{i2}, \dots, x_{ip})^T$ is the input vector of real valued (crisp) [3] .

The linear programming formula of the fuzzy regression problem can be written as follows [3]:

$$Minimize\ S = \sum_{i=1}^{n} \sum_{j=0}^{p} c_j |x_{ij}| \qquad (3)$$

Subject to:

$$\sum_{j=0}^{p} \alpha_j x_{ij} + (1 - \square) \sum_{j=0}^{p} c_j |x_{ij}| \geq y_i + (1 - \square) e_i \qquad (4)$$

$$\sum_{j=0}^{p} \alpha_j x_{ij} - (1 - \square) \sum_{j=0}^{p} c_j |x_{ij}| \leq y_i - (1 - \square) e_i \qquad (5)$$

$$c_j \geq 0, \alpha_j \in R, \quad x_{i0} = 1, \quad i = 1,2, \dots, n, \quad 0 \leq \quad \leq 1 \qquad (6)$$

The h value (degree of confidence) is selected by the decision maker, where h is belongs to [0, 1].

2. Savic & Pedrycz approach

Savic and Pedrycz formulated the fuzzy regression by combining the ordinary least squares with minimum fuzziness criterion. The method is constructed in two successive steps. The first step employs ordinary least square regression to obtain fuzzy regression parameters. The minimum fuzziness criterion is used in the second step to find the spread of fuzzy regression parameters.

In the first step, the available information about the value of the center of the fuzzy observations is used to fit a regression line to the data.

In fact, the fuzzy data are regressed as simplified crisp data and the regression analysis is conducted as it is an ordinary least squares regression. The results of this step are employed as center values of the fuzzy regression parameters.

In the next step, the minimum fuzziness criterion is used to determine fuzzy parameters. Spreads of the fuzzy parameters are obtained by equation (4), (5) as the minimum fuzziness method with the distinction of employing the fuzzy centers of regression parameters resulting from the first step [4].

3. First Proposed method

Our first proposed method is a modification of Savic & Pedrycz method to deal with case of multi-collinearity among the crisp explanatory variables. The method may be summarized as follows:

In the first step the principle component regression is used instead of ordinary least squares regression to determine fuzzy center values of fuzzy regression coefficients. In the second step, the minimum fuzziness criterion is used to find the spread of fuzzy regression coefficients.

4. Second Proposed method

Also this proposed method is deal with case of multi-collinearity among the crisp explanatory variables and it is a modification of Savic-Pedrycz method.

This method summarized as follows:

The ridge regression is used instead of ordinary least squares regression to determine fuzzy center values of fuzzy regression coefficients in the first step.

The minimum fuzziness criterion is used in the second step to find the spread of fuzzy regression coefficients.

Numerical example

Numerical example is used in this section to illustrate the different approaches that are summarized in previous sections. Data used in the experiment consist of 54 observations taken from transportation laboratory of the Civil Engineering Department of the University of Baghdad which is illustrated in the following table:

TABLE 1: Fatigue Test Results

	Y=Ln Nf	X1= Ln [illegible]	X2=Ln [illegible]Pa)	X3=Ln [illegible]MPa)	X4=Ln [illegible]
1	0.795855349	-8.111728083	8.538954683	-0.494296322	6.081
2	0.831788925	-8.111728083	8.559869466	-0.494296322	3.822
3	0.70662007	-8.517193191	8.74687532	-0.798507696	5.984
4	0.866475944	-8.111728083	8.724532511	-0.494296322	5.716
5	0.843485463	-8.111728083	8.714239144	-0.494296322	3.95
6	0.102663757	-7.824046011	8.543445563	-0.494296322	2.114
7	0.28092649	-7.824046011	8.350429974	-0.494296322	6.57
8	0.432167891	-7.824046011	8.285765421	-0.494296322	6.873
9	0.563511756	-7.60090246	8.378160983	-0.198450939	4.589
10	0.70662007	-7.824046011	8.330863613	-0.494296322	6.962
11	0.359403715	-7.60090246	8.547722396	-0.198450939	2.68
12	0.320934739	-7.60090246	8.298539545	-0.198450939	6.584
13	0.260305654	-8.517193191	8.753371421	-0.798507696	3.05
14	0.795855349	-8.517193191	8.77971129	-0.798507696	2.404
15	0.217742741	-8.111728083	8.428361978	-0.798507696	5.46
16	0.499994677	-8.111728083	8.541885804	-0.798507696	6.345
17	0.63762373	-8.517193191	8.708639656	-0.798507696	7.7
18	0.758582267	-8.517193191	8.396154863	-0.798507696	6.291
19	0.888949719	-8.517193191	8.697345731	-0.798507696	6.36
20	0.48346438	-8.111728083	8.777709596	-0.494296322	2.615
21	0.623234239	-8.517193191	8.76623838	-0.798507696	2.61
22	0.745842651	-8.111728083	8.402679805	-0.798507696	2.823
23	0.877775964	-8.517193191	8.56674497	-0.798507696	2.825
24	0.679590067	-8.111728083	8.803874764	-0.494296322	2.829
25	0.665796052	-7.824046011	8.377471248	-0.494296322	2.827
26	0.51625616	-8.111728083	8.582044164	-0.798507696	2.829

27	0.608634663	-8.111728083	8.29529886	-0.798507696	2.828
28	0.819953958	-8.517193191	8.730690366	-0.798507696	2.61
29	0.771161626	-8.111728083	8.427487278	-0.798507696	2.829
30	0.578780077	-8.517193191	8.649974303	-0.798507696	2.82
31	0.732938639	-8.111728083	8.362642432	-0.798507696	2.828
32	0.466656233	-8.517193191	8.528528701	-0.798507696	2.82
33	0.340354198	-7.824046011	8.321664807	-0.494296322	2.826
34	0.466656233	-7.824046011	8.426392827	-0.494296322	2.82
35	0.855046772	-8.111728083	8.648396877	-0.494296322	7.26
36	0.651809098	-7.824046011	8.514990768	-0.494296322	6.86
37	0.783584708	-7.418580903	8.610683535	-0.198450939	7.05
38	0.320934739	-7.60090246	8.607216694	-0.198450939	6.67
39	0.807977244	-7.418580903	8.239857411	-0.198450939	7.67
40	0.693196393	-8.111728083	8.701180028	-0.494296322	6.44
41	0.532257434	-7.824046011	8.160232492	-0.494296322	7.87
42	0.831788925	-7.418580903	8.222553638	-0.198450939	6.5
43	0.150295937	-7.60090246	8.640295389	-0.198450939	6.49
44	0.320934739	-7.60090246	8.644002038	-0.198450939	7.45
45	0.026670837	-7.418580903	8.30474227	-0.198450939	7.55
46	0.126763423	-7.824046011	8.58634605	-0.198450939	7.28
47	0.026670837	-7.418580903	8.538563217	-0.198450939	6.9
48	0.239250631	-8.111728083	8.50512061	-0.494296322	6.35
49	0.37809712	-7.60090246	8.494538501	-0.198450939	3.54
50	0.102663757	-7.418580903	8.183118079	-0.198450939	7.495
51	0.150295937	-7.824046011	8.563885919	-0.198450939	7.33
52	0.077968924	-7.418580903	8.335431478	-0.198450939	7.32
53	0.195762075	-7.60090246	8.478452363	-0.198450939	6.485
54	0.026670837	-7.60090246	8.318742253	-0.198450939	7.316

TABLE 2: The Name and description of variables

$Y = N_f$ (cycle)	The fatigue life	dependent variable
$x_1 = \varepsilon_0$ (mm/mm)	Initial tensile strain	the first independent variable
$x_2 = S_0$ (MPa)	Initial flexural stiffness	the second independent variable
$x_3 = \sigma$ (MPa)	stress level	the third independent variable
$x_4 = Av$ (%)	air void	the fourth independent variable

In order to determine the linear relationship among these variables, Pearson correlations were calculated in the following table:

TABLE 3: The Pearson Correlation

	X1	X2	X3	X4	Y
X1	1	-0.583	0.907	0.518	-0.584
X2	-0.583	1	-0.316	-0.236	0.438
X3	0.907	-0.316	1	0.546	-0.597
X4	0.518	-0.236	0.546	1	-0.264
Y	-0.584	0.438	-0.597	-0.264	1

In order to checks the existence of multi-collinearity problem we use the Farrar-Glober test as follows [5]:
The hypothesis to be tested is:
$H_0: x_j$'s are orthogonal.
Against:
$H_1: x_j$'s are not orthogonal

$$x_0^2 = -\left[n - 1 - \frac{1}{6}(2k+5)\right] Ln|D| = -\left[54 - 1 - \frac{1}{6}(2 \times 4 + 5)\right] Ln|0.0333| = 55.8970$$

The theoretical value of x_0^2 with $\frac{k(k-1)}{2} = 6$ degrees of freedom and 0.05 level of significant is found to be, since the calculated value is:
More than the theoretical value we reject the null hypothesis H_0 and we conclude that the problem of multi-collinearity exists.

Hence, we reject H_0 and we conclude that the problem of multi-collinearity is existing.
As shown in above table, there is at least one value greater than 10 and that is an indicator of the existence of multi-collinearity problem.
The regression equation obtained by applying Savic & Pedrycz method and assuming that h=0.5 is shown as follows:

$$\hat{y}_i = (-1.49500932075108, 0.001121256946788) + (-0.213169442048952, 0.000159877088712)x_{1j} + (0.022168081623166, 0.000016626047275)x_{2j} + (-0.252314818567323, 0.000189236132903)x_{3j} + (-0.001159435826688, 0.000000869575698)x_{4j}$$

The regression equation obtained by applying the first proposed method and assuming that h=0.5 is shown as follows:

$$\hat{y}_i = (-1.03, 0.000946878879525234) + (-0.1752, 0.000145638563866216)x_{1j} + (0.001, 0.000001688059099767)x_{2j} + (-0.3, 0.000207118003587208)x_{3j} + (-0.00231, 0.000001301040871456)x_{4j}$$

The regression equation obtained by applying the second proposed method and assuming that h=0.5 is shown as follows:

$$\hat{y}_i = (-0.261, 0.000658503653435002) + (-0.1804, 0.000147588543474805)x_{1j} + (-0.09308, 0.000026591951252957)x_{2j} + (-0.3037, 0.000208505555430326)x_{3j} + (-0.0045, 0.000002122291449845)x_{4j}$$

To clarify the efficiency of proposed approaches, width values were calculated to each of the mentioned methods. The upper limits of savic and pedrycz was obtained by the following equation:

$$T\square e\ upper\ limit\ is: Y_i{}^U = \sum_{j=0}^{n} (a_j + b_j)x_{ji} \quad , \quad \forall i \ ,$$

$$w\square ere\ a_j\ are\ t\square e\ center\ of\ parameters\ and\ b_j\ are\ st\square e\ spread\ of\ t\square e\ paraneters$$

$$y^U = (-1.495009320751 + 0.001121256947) + (-0.213169442049, +0.000159877089)x_{1j} + (0.022168081623 + 0.000016626047)x_{2j} + (-0.252314818567 + 0.000189236133)x_{3j} + (-0.001159435827 + 0.000000869576)x_{4j}$$

Similarly, lower limits for savic and pedrycz model were calculated by the following equation:

$$lower\ limit: Y_i{}^L = \sum_{j=0}^{n} (a_j - b_j)x_{ji} \quad , \quad \forall i \ ,$$

$$w\square ere\ a_j\ are\ t\square e\ center\ of\ parameters\ and\ b_j\ are\ st\square e\ spread\ of\ t\square e\ parameterers$$

$$y^L = (-1.495009320751 - 0.001121256947) + (-0.213169442049 - 0.000159877089)x_{1j} + (0.022168081623 - 0.000016626047)x_{2j} + (-0.252314818567 - 0.000189236133)x_{3j} + (-0.001159435827 - 0.000000869576)x_{4j}$$

The upper and lower limits for the first proposed approach were calculated as follows:

$$y^U = (-1.03 + 0.0009468788795) + (-0.1752 + 0.0001456385639)x_{1j} + (0.001 + 0.0000016880591)x_{2j} + (-0.3 + 0.0002071180036)x_{3j} + (-0.00231 + 0.0000013010409)x_{4j}$$

$$y^L = (-1.03 - 0.0009468788795) + (-0.1752 - 0.0001456385639)x_{1j} + (0.001, -0.0000016880591)x_{2j} + (-0.3 - 0.0002071180036)x_{3j} + (-0.00231 - 0.0000013010409)x_{4j}$$

Also, the upper and lower limits for the second proposed approach were calculated as follows:

$$y^U = (-0.261 + 0.0006585036534) + (-0.1804 + 0.0001475885435)\ x_{1j} + (-0.09308 + 0.0000265919513)\ x_{2j} + (-0.3037 + 0.0002085055554)\ x_{3j} + (-0.0045 + 0.0000021222914)\ x_{4j}$$

$$y^L = (-0.261 - 0.0006585036534) + (-0.1804 - 0.0001475885435)x_{1j} + (-0.09308 - 0.0000265919513)x_{2j} + (-0.3037 - 0.0002085055554)x_{3j} + (-0.0045 - 0.0000021222914)x_{4j}$$

The width for each models computed in MS Excel are shown as follows:

TABLE 5: Comparison among Savic & Pedrycz, First proposed and second proposed methods

	Savic & Pedrycz (using Least Square method)			First proposed method (using principle component)			second proposed method (using Ridge regression)		
	the upper limit	the lower limit	Width	the upper limit	the lower limit	width	the upper limit	the lower limit	width
1	0.541001326	0.541245135	-0.000243809	0.533641922	0.534269080	-0.000627158	0.529988584	0.530617691	-0.000629107
2	0.544082516	0.544329558	-0.000247042	0.538874280	0.539515132	-0.000640852	0.538204432	0.538839346	-0.000634914
3	0.708793782	0.709275630	-0.000481848	0.696152733	0.697224050	-0.001071317	0.676484897	0.677357672	-0.000872775
4	0.545541193	0.545779465	-0.000238273	0.534671340	0.535297118	-0.000625778	0.514357338	0.514986768	-0.000629430
5	0.547358866	0.547600552	-0.000241686	0.538734891	0.539371900	-0.000637009	0.523260130	0.523894190	-0.000634060
6	0.484417955	0.484576525	-0.000158571	0.492423409	0.493020317	-0.000596908	0.495560971	0.496116590	-0.000555619
7	0.474973389	0.475130628	-0.000157239	0.481949044	0.482521932	-0.000572888	0.493480334	0.494025009	-0.000544676
8	0.473187779	0.473346642	-0.000158863	0.481184758	0.481757029	-0.000572271	0.498136095	0.498680201	-0.000544105
9	0.355761813	0.355738255	0.000023558	0.358899936	0.359081991	-0.000182055	0.369801535	0.370163725	-0.000362190
10	0.474085157	0.474242365	-0.000157208	0.481024988	0.481595814	-0.000570826	0.493538048	0.494081769	-0.000543722
11	0.361735186	0.361709310	0.000025876	0.363474999	0.363665631	-0.000190633	0.362607061	0.362973646	-0.000366585
12	0.351684095	0.351659715	0.000024380	0.354217309	0.354388476	-0.000171167	0.368237659	0.368594927	-0.000357268
13	0.712337129	0.712823864	-0.000486735	0.702927672	0.704017182	-0.001089510	0.689079433	0.689959821	-0.000880388
14	0.713669906	0.714156888	-0.000486983	0.704444515	0.705537540	-0.001093025	0.689533922	0.690415902	-0.000881980
15	0.615966799	0.616330501	-0.000363702	0.626041108	0.627044553	-0.001003445	0.635402045	0.636159156	-0.000757111
16	0.617459961	0.617818348	-0.000358388	0.624114165	0.625109845	-0.000995680	0.620854090	0.621608515	-0.000754425
17	0.705957435	0.706437571	-0.000480135	0.692155518	0.693216873	-0.001061355	0.672324040	0.673192479	-0.000868439

18	0.700657471	0.701150448	-0.000492976	0.695090343	0.696166658	-0.001076315	0.707748264	0.708621424	-0.000873160
19	0.707259362	0.707742203	-0.000482841	0.695235329	0.696305274	-0.001069945	0.679403516	0.680275480	-0.000871964
20	0.550313625	0.550555522	-0.000241897	0.541878669	0.542522763	-0.000644094	0.523358170	0.523995490	-0.000637319
21	0.713132348	0.713619421	-0.000487072	0.703955692	0.705047696	-0.001092004	0.689861226	0.690742715	-0.000881489
22	0.618452187	0.618821328	-0.000369142	0.632098409	0.633118828	-0.001020418	0.649655567	0.650419627	-0.000764060
23	0.708457554	0.708950886	-0.000493332	0.703258253	0.704352848	-0.001094595	0.707462516	0.708344119	-0.000881603
24	0.550646158	0.550886814	-0.000240655	0.541411421	0.542053662	-0.000642241	0.519960039	0.520596713	-0.000636674
25	0.479909805	0.480072655	-0.000162850	0.490610995	0.491206723	-0.000595729	0.507802008	0.508356332	-0.000554324
26	0.622424381	0.622787548	-0.000363167	0.632265701	0.633282545	-0.001016844	0.632933643	0.633697082	-0.000763438
27	0.616064179	0.616436883	-0.000372704	0.631978435	0.633000940	-0.001022505	0.659627911	0.660392320	-0.000764409
28	0.712343726	0.712831980	-0.000488254	0.703919794	0.705012499	-0.001092705	0.693169975	0.694051584	-0.000881609
29	0.618995582	0.619363888	-0.000368306	0.632109620	0.633129512	-0.001019892	0.647319537	0.648083498	-0.000763960
30	0.710309765	0.710800338	-0.000490573	0.703353838	0.704446823	-0.001092985	0.699738163	0.700619498	-0.000881335
31	0.617558177	0.617928641	-0.000370464	0.632046443	0.633067620	-0.001021177	0.653359685	0.654123867	-0.000764182
32	0.707615530	0.708110141	-0.000494611	0.703231195	0.704326575	-0.001095380	0.711042115	0.711923860	-0.000881745
33	0.478672914	0.478837621	-0.000164707	0.490556945	0.491153780	-0.000596835	0.513000876	0.513555391	-0.000554515
34	0.481003226	0.481164461	-0.000161235	0.490676547	0.491271354	-0.000594808	0.503279961	0.503834137	-0.000554177
35	0.542063320	0.542301439	-0.000238119	0.531032635	0.531650272	-0.000617638	0.514497923	0.515123593	-0.000625669
36	0.478288138	0.478439401	-0.000151263	0.481446232	0.482014065	-0.000567833	0.476858670	0.477402035	-0.000543365
37	0.319232791	0.319138923	0.000093868	0.321532196	0.321659483	-0.000127288	0.304223174	0.304525069	-0.000301895
38	0.358432370	0.358397577	0.000034794	0.354330638	0.354495181	-0.000164544	0.339119623	0.339475625	-0.000356002
39	0.310287810	0.310205195	0.000082615	0.319727449	0.319858177	-0.000130728	0.335949850	0.336251384	-0.000301534
40	0.544184323	0.544422113	-0.000237790	0.532977578	0.533599295	-0.000621717	0.513273890	0.513901515	-0.000627625
41	0.469247777	0.469409080	-0.000161303	0.478758030	0.479326551	-0.000568521	0.505335286	0.505877220	-0.000541935
42	0.311259454	0.311179449	0.000080005	0.322409022	0.322547397	-0.000138376	0.342823934	0.343128571	-0.000304637
43	0.359374754	0.359339173	0.000035580	0.354779280	0.354944296	-0.000165015	0.336850480	0.337206838	-0.000356359
44	0.358344761	0.358307388	0.000037373	0.352568421	0.352727368	-0.000158948	0.332186720	0.332540568	-0.000353848
45	0.311866290	0.311781726	0.000084564	0.320069799	0.320199997	-0.000130198	0.330450321	0.330751948	-0.000301627
46	0.404794347	0.404830537	-0.000036191	0.391975811	0.392179567	-0.000203756	0.378540619	0.378960101	-0.000419482
47	0.317806607	0.317715399	0.000091209	0.321805396	0.321935042	-0.000129647	0.311610816	0.311913345	-0.000302529
48	0.539939073	0.540183539	-0.000244466	0.532987205	0.533613350	-0.000626145	0.531927652	0.532556173	-0.000628521
49	0.359558950	0.359533347	0.000025603	0.361437376	0.361623687	-0.000186311	0.363688447	0.364052974	-0.000364527
50	0.309231814	0.309151390	0.000080424	0.320073854	0.320206794	-0.000132940	0.342018324	0.342320505	-0.000302181
51	0.404238147	0.404274997	-0.000036851	0.391837786	0.392041672	-0.000203886	0.380406235	0.380825663	-0.000419427
52	0.312813591	0.312728407	0.000085185	0.320631373	0.320762402	-0.000131029	0.328628522	0.328930644	-0.000302122
53	0.355790106	0.355759916	0.000030190	0.354627376	0.354795614	-0.000168238	0.351937049	0.352293967	-0.000356918
54	0.351284215	0.351257891	0.000026325	0.352549076	0.352715274	-0.000166198	0.363064178	0.363419473	-0.000355295

And the average widths of them are:

TABLE 6: The Result of Average Width among Savic & pedrycz, First proposed and Second proposed

Average width	Savic & pedrycz (using Least Square method)	First proposed method (using Principle Component)	Second proposed method (using Ridge regression)
	-0.000199068573	-0.0006079500772	-0.0005887454820

The results show that the first proposed approach has less average width followed by the second proposed approach.

CONCLUSION

1. According to the above explained results through the application of Savic & Pedrycz and the first and second proposed methods, it was found that the best method to minimize the spreads applying the linear programing to data that suffer from linear multi co-linearity is the first proposed approach followed by the second proposed approach.
2. Average width results clearly explain the first and second proposed approaches are capable to deal with cases in which the data suffers from linear multi co-linearity problem.

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